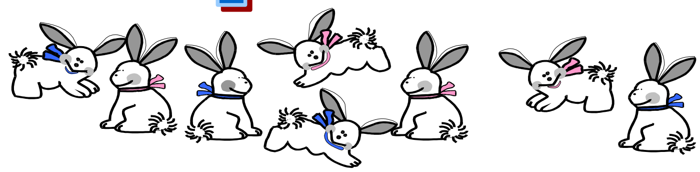
# Chaotic Population



The way in which populations change can be modelled using a recurrence relation.

In the following recurrence relation, the population in a generation $P_{t+1}$ is related to the population in the previous generation, $P_t$, by:

$$P_{t+1} = kP_t(1 - P_t) \qquad \text{where } k \text{ is a positive constant.}$$

This recurrence relation consists of two terms:

- a growth term, $kP_t$, that increases as $P_t$ increases
  ($k$ can be taken to depend on the amount of food available)
- a death term, $kP_t^2$, that also increases as $P_t$ increases
  (this could be due to overcrowding and/or competition for the available food).

The sequence of values $P_1$, $P_2$, $P_3$, ….will depend on the initial population $P_0$ and $k$.

In this activity you will investigate the effect of different values of $k$ and $P_0$.

**Using a graphic calculator**

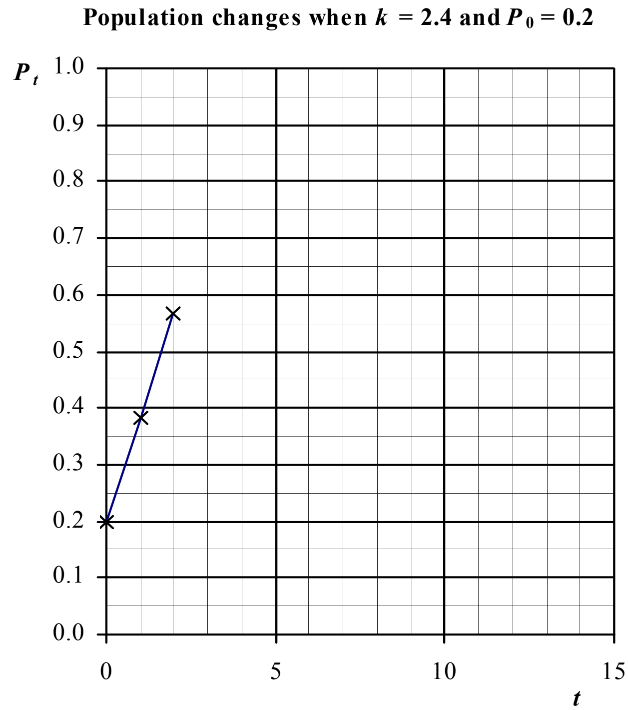
First take $k$ to be 2.4 and $P_0$ to be 0.2 (assume this is in millions or thousands if you wish).

- Enter the value of $P_0$, in this case 0.2, into your calculator.
- Now enter the recurrence relation
  $P_{t+1} = kP_t(1 - P_t)$
  using 2.4 as $k$ and the Ans key as $P_t$ :
  2.4Ans(1 – Ans)
- Repeatedly press the 'equals' key to give successive terms in the sequence. Check that the values you get agree with those given in the table when rounded to 3dp.
- Complete the table.

| $t$ | $P_t$ |
|---|---|
| 0 | 0.2 |
| 1 | 0.384 |
| 2 | 0.568 |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| 9 | |
| 10 | |
| 11 | |
| 12 | |
| 13 | |
| 14 | |
| 15 | |

The way in which the population changes can be shown on a graph.

- Complete this graph.
- What happens to the population in the case when $k = 2.4$ and $P_0 = 0.2$?
- Try different values of $P_0$ with $k = 2.4$
  Does the same thing happen each time?




**Using a spreadsheet**
Working out values and plotting graphs is much quicker using a spreadsheet.
A spreadsheet has been set up to allow you to investigate other values of $k$ quickly and easily.
Follow the instructions given on the spreadsheet and write a summary of what you find.

**Chaos**
You should have found that for some values of $k$, the population behaves chaotically.
This is similar to the way in which climate can behave.

Chaos was first discovered by the meteorologist Edward Lorenz who was using a computer program to simulate weather. After carrying out one simulation he wanted to check his results, but to save time he decided not to run the whole simulation again. Instead he entered some of the data from the middle of his previous simulation, expecting the remainder of the sequence to be as before. To his surprise, the new pattern in the results was very different. He discovered that the problem had arisen because of small inaccuracies in the data he had used – the original values stored in the computer were calculated to 6 decimal places, but then printed to 3 decimal places. Entering these slightly different values had caused a significant difference in the resulting pattern of values. This effect is often called the "butterfly effect"after Ian Stewart wrote the following in his book "Does God play dice? The mathematics of Chaos":

***The flapping of a single butterfly's wing today produces a tiny change in the state of the atmosphere. Over a period of time, what the atmosphere actually does diverges from what it would have done. So, in a month's time, a tornado that would have devastated the Indonesian coast doesn't happen. Or maybe one that wasn't going to happen, does.*** (P141)